

A ROBUST CONTROL SCHEME FOR FLEXIBLE ARMS WITH FRICTION IN THE JOINTS

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ABSTRACT

A general control scheme to control flexible arms with friction in the joints is proposed in this paper. This scheme presents the advantage of being robust in the sense that it minimizes the effects of the Coulomb friction existing in the motor and the effects of changes in the dynamic friction coefficient. A justification of the robustness properties of the scheme is given in terms of the sensitivity analysis.

1 INTRODUCTION

In the last few years, considerable research effort has been devoted in controlling flexible structures and, in particular, flexible arms [1-6]. Very little effort has been made in controlling flexible arms when Coulomb and dynamic friction are present in the joints, in spite of this being common in practice. In fact we have not found any paper that specifically deals with and tries to solve this problem.

This paper is devoted to solving the above problem. In order to do this, a new general control scheme is proposed. Existing methods to control flexible arms are based on the explicit control of the tip position, where the controller generates the current for the D.C. motor of the joint as a control signal. The proposed method is based on the simultaneous control of the joint motor position and tip position, and the implementation of two nested closed loops: the inner loop that controls the motor position and the outer loop that controls the tip position.

The general control scheme is presented in Section 2. Section 3 compares the sensitivity of our general control scheme with the sensitivity of other existing methods. This shows our scheme to be more robust to changes in friction than the others. Finally, some conclusions are drawn in Section 4.

2 GENERAL CONTROL SCHEME

As was mentioned in the introduction, there are many applications in which friction must be taken into account when controlling a flexible arm. Only in the case when the coupling torque between the beam and the motor is many times larger than the friction torque it can be neglected. This may be true for very large structures, but it is not true in many industrial applications.

In order to reduce the effects of friction, the control scheme of figure 1 is proposed. In this scheme two variables are controlled: motor and tip position (θ_m and θ_t respectively). These two variables are controlled by two nested closed loops and two different controllers ($R_1(s)$ and $R_2(s)$) are used. These controllers are each designed separately and according to different criteria. In figure 1, the open-loop transfer function of the motor $G_M(s)$ has all its poles and zeroes in the left half-plane. The open-loop transfer function of the flexible beam $G_B(s)$ has its poles in the left half-plane but may have (for arms with more than one vibrational mode) some zeroes in the right half-plane (non-minimum phase system). $F(s)$ is a filter for the reference and is normally designed

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in conjunction with $R_1(s)$.

The use of this control scheme has been motivated by the well known property that the sensitivity of a closed-loop system to perturbations can be made arbitrarily small by increasing the gain of the open-loop, provided that the system remains stable (Kuo [8]). Therefore considering the friction (in general terms) as a perturbation, we can reduce its effects by increasing the gains of the controller.

If we try to do this using the existing control schemes (such as shown in figure 2) for flexible arms, the gains cannot be arbitrarily increased because of the right half-plane zeroes. In the proposed scheme, because $G_M(s)$ is minimum phase, the gains of the inner loop can be arbitrarily increased (using an appropriate controller $R_2(s)$) without making the system unstable. Intuitively, the high gain inner loop to control the motor position makes the system insensitive to friction and, a second outer loop may then be designed to control the tip position. This second loop cannot have a high gain because $G_B(s)$ is non-minimum phase. However, the friction effects have been largely removed by the high-gain inner loop controlling the motor position.

3 SENSITIVITY ANALYSIS

The previous ideas intuitively justify the reason for using our control scheme. This section is devoted to giving an analytical proof. The analysis carried out here is quite straightforward and will give a quantitative idea of how much the robustness is increased using the nested loop scheme. In order to do this comparison, a typical control scheme like the one shown in figure 2 will be used (Cannon [1]). The sensitivity characteristics of this system will be taken as representative of the existing methods because they are based on controlling the tip position using a controller that generates a command for the current of the D.C. motor. The sensitivities of these methods are thus of the same order of magnitude. Two comparative analyses will be done : one checking the signal-to-noise ratio (considering the Coulomb friction as the noise), and the other checking the sensitivity to variations in the dynamic friction coefficient.

In order to do this comparative analysis, the state-space control scheme of figure 2 is first expressed in terms of its equivalent transfer functions.

Assume that the plant is represented by the state space-equations

$$\begin{aligned}\dot{\underline{x}} &= A\underline{x} + Bi \\ \theta_t &= C\underline{x}\end{aligned}\quad (1)$$

where $C = [1 \ 0 \ 0 \ 0 \ \dots \ 0]$ is of dimension n . The controller is a row vector of dimension n of the form

$$\underline{K}^T = [k_1 \mid \hat{\underline{K}}^T] \quad (2)$$

where $\hat{\underline{K}}$ is a vector of dimension $n-1$, and k_1 is the coefficient corresponding to the tip position error (error in the first state). The states \underline{x} of the system can be reconstructed from the state-space equation (1) as follows

$$\begin{pmatrix} \theta_t \\ \dot{\theta}_t \\ \ddot{\theta}_t \\ \vdots \\ \theta_t^{(n-1)} \end{pmatrix} = \begin{pmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ CB & 0 & 0 & \dots & 0 & 0 \\ CAB & CB & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ CA^{n-2}B & CA^{n-3}B & CA^{n-4}B & \dots & CB & 0 \end{pmatrix} \begin{pmatrix} i \\ \dot{i} \\ \ddot{i} \\ \vdots \\ i^{(n-1)} \end{pmatrix} \quad (3)$$

where $[C \ CA \ CA^2 \ \dots \ CA^{n-1}]$ is the observability matrix. If the system is observable (which is true in all the models of flexible arms), then the states \underline{x} of the system may be reconstructed from the measurements of the input i and the output θ_t using a linear law of the form

$$\underline{x}(s) = P(s)\theta_t(s) + Q(s)i(s) \quad (4)$$

where P and Q are polynomial column vectors in s . Equation (4) can be easily obtained from equation (3) by inverting the observability matrix. Adding equation (4) in the scheme of figure 2 and substituting the state-space equation (1) of the plant by $G_M(s)G_B(s)$, we obtain the system shown in figure 3. Figure 3 can be simplified and an equivalent transfer function scheme can be obtained, as shown in figure 4, where

$$F(s) = \frac{k_1 G_M(s)G_B(s)}{KP(s)G_M(s)G_B(s) + KQ(s)} \quad (5)$$

and

$$R(s) = \frac{KP(s)G_M(s)G_B(s) + KQ(s)}{G_M(s)G_B(s)} \quad (6)$$

The comparative analysis may now be done between the schemes of figures 1 and 4.

3.1 Signal-to-Noise Ratio Analysis

The signal-to-noise ratio of a system is defined (Kuo [8]) as

$$R = \frac{\text{Output due to signal}}{\text{Output due to noise}} \quad (7)$$

and is a measure of the sensitivity of the system to perturbation signals (in this case the Coulomb friction). The comparison of the ratios of the schemes of figures 1 and 4 is done here for the same levels of input θ_t^r and perturbation p (which is added to the current of the motor).

The output of the nested double loop scheme in terms of the reference input θ_t^r and the noise p can be written from figure 1 as

$$\theta_t = \frac{G_M(s)G_B(s)}{1 + G_M(s)R_2(s)(1 + G_B(s)R_1(s))} (R_1(s)R_2(s)F(s)\theta_t^r + p) \quad (8)$$

The signal-to-noise ratio for this scheme is given by

$$R_{\theta_t^r, p} = R_1(s)R_2(s)F(s) \quad (9)$$

Similarly the output and the signal-to-noise ratio for the system shown in figure 4 can be written as

$$\theta_t = \frac{G_M(s)G_B(s)}{1 + G_B(s)G_M(s)R(s)} (R(s)F(s)\theta_t^r + p) \quad (10)$$

$$R_{\theta_t^r, p} = R(s)F(s) = k_1 \quad (11)$$

Comparing both results, expression (9) can be made large by designing the controller $R_2(s)$ with arbitrarily high gain because the inner loop is minimum phase. However, k_1 in expression (11), and all the parameters of $R(s)$ in general, are limited by the stability margin since the system is non-minimum phase. The gains of $R_1(s)$ in equation (9) are also limited for the same reason.

From all this it follows that, in general, (9) may be made larger than (11) by properly choosing the gains of the controller of the inner loop. It must be mentioned here that this is a theoretical analysis. In practice, the gains of the inner loop will be limited by the saturation of the amplifier, instability because of unmodelled high frequency dynamics, or even instability because of the discretization of the signals when using digital controllers. But in any case these limits are much larger than the ones imposed by the non-minimum phase characteristic.

3.2 Sensitivity to the Dynamic Friction Coefficient

Dynamic friction is the second manifestation of friction. This is normally assumed to be linear and is included in the model of the plant. In many cases, however, this assumption of linearity is not correct. Often, the dynamic friction coefficient changes noticeably depending on the sense of rotation of the motor [9] or on the position of the rotor of the motor relative to the stator (con-

fronted poles), etc. It will be shown here (by performing the sensitivity analysis of both systems to changes in the dynamic friction coefficient) that the robustness of the system due to changes in the dynamic friction coefficient may be significantly improved by using the nested double loop scheme.

The sensitivity to changes in the parameter v of a system whose closed-loop transfer function is $M(s)$, is defined (Kuo [8]) by

$$S_{M,v} = \frac{dM(s)/M(s)}{dv/v} \quad (12)$$

In order to do this analysis, let us express $G_M(s)$ in the form

$$G_M(s) = \frac{K_m}{Js^2 + vs + T(s)} \quad (13)$$

which is the typical transfer function of a D.C. motor, except the term $T(s)$ which represents the coupling torque between the beam and the motor. This allows us to characterize the influence of the dynamic friction coefficient, v , in the general transfer function.

Performing some calculations, we find the sensitivity to v for the closed-loop system of figure 1 is given by

$$S_{M,v1} = \frac{-sv}{[1 + R_2(s)G_M(s)\{1 + R_1(s)G_B(s)\}](Js^2 + vs + T(s))} \quad (14)$$

The sensitivity for the system of figure 4 is

$$S_{M,v2} = \frac{-sv}{(1 + R(s)G_M(s)G_B(s))(Js^2 + vs + T(s))} \quad (15)$$

The ratio of the two sensitivities given by equations (14) and (15) can be written as

$$\frac{S_{M,v1}}{S_{M,v2}} = \frac{1 + G_M(s)[R(s)G_B(s)]}{1 + G_M(s)[R_2(s)(1 + R_1(s)G_B(s))]} \quad (16)$$

The ratio given by equation (16) is significantly smaller than 1 since $R_2(s)(1 + R_1(s)G_B(s)) \gg R(s)G_B(s)$. Notice also that the gains of $R(s)$ and $R_1(s)$ are bounded by a stability margin and will thus have the same order of magnitude; but the gain of $R_2(s)$ may be increased arbitrarily. Consequently, scheme of figure 1 is more robust in general to changes in the dynamic friction than the scheme of figure 4, by a factor of approximately $R_2(s)$.

3.3 Comparison of the Characteristic Equations

The previous analysis gives a quantitative justification of how the robustness of the system is increased using the two nested loops scheme. A qualitative understanding of how the nested loops modify the stability of the system allowing higher gains may be obtained by looking at the characteristic equations of the two systems.

In the traditional control scheme, the robustness depends on $R(s)$. The characteristic equation of the system shown in figure 4 is given by

$$1 + R(s)G_M(s)G_B(s) = 0 \quad (17)$$

Substituting equation (6) in (17), the characteristic equation can be expressed in terms of the feedback gains as

$$1 + \frac{KG_M(s)G_B(s)P(s)}{1 + KQ(s)} = 0 \quad (18)$$

Notice that in equation (18) the right half-plane zeroes of $G_B(s)$ limit the gain, K , and consequently the gains of $R(s)$.

In the proposed scheme, we note from equation (9) that the signal-to-noise robustness depends on the product $R_1(s)R_2(s)$, while from equation (14), that the sensitivity depends both on this product and also on the $R_2(s)$ term. The characteristic equation of the system can now be written as

$$1 + R_2(s)G_M(s)(1 + G_B(s)R_1(s)) = 0 \quad (19)$$

If the factor $1 + G_B(s)R_1(s)$ had all its zeroes in the left half-plane, then the gains of $R_2(s)$ could be made arbitrarily large. Because $G_B(s)$ is stable, there always exists an $R_1(s)$ of moderately low gains that makes the above factor have all its zeroes in the left half-plane (this may be justified from the root locus plot of $G_B(s) R_1(s)$) and, consequently, the product $R_1(s)R_2(s)$ may be arbitrarily large, improving the robustness of the system.

4 CONCLUSIONS

In this paper, a new control scheme to control flexible arms with friction in the joints has been presented. The proposed scheme is based on a nested double loop structure. It has been shown to be more robust to changes in the dynamic friction coefficient as well as less sensitive to the presence of Coulomb friction as compared to existing methods.

The non-minimum phase feature that appears in the most of the flexible arms precludes the use of very high gains in the controller, which is the classical way of increasing the robustness. The proposed scheme divides the control problem into two: the first consists of the design of an inner loop controller for a minimum phase system, and the second consists of the design of an outer loop to control the tip position where the non-minimum phase feature must be considered.

Analytical comparative studies of the sensitivity to both kinds of perturbation have shown that the proposed scheme is more robust to these perturbations than traditional schemes by a ratio of roughly the gain of the controller of the inner loop. This gain may be made arbitrarily high (the only limitation will be the saturation of the amplifier of the motor and the presence of high frequency unmodelled modes).

REFERENCES

1. Cannon, Robert H and Schmitz, E., "Precise Control of Flexible Manipulators", Robotics Research, 1985.
2. Harahima, F. and Ueshiba, T., "Adaptive Control of Flexible Arm using the End-Point Position Sensing", Proceedings Japan-USA Symposium of Flexible Automation, Osaka (Japan), July 1986.
3. Siciliano, B., Yuan, B.S. and Book, W.J., "Model Reference Adaptive Control of a One Link Flexible Arm", 25th IEEE Conference on Decision and Control, Athens, December 1986.
4. Ower, J. C. and Van de Vegte, J., "Classical Control Design for a Flexible Manipulator: Modeling and Control System Design", IEEE Journal of Robotics and Automation, Vol. RA-3, No. 5, October 1987.
5. Rovner, D.M. and Cannon, R.H., "Experiments Towards on-line Identification and Control of a Very Flexible One-Link Manipulator", International Journal of Robotics Research, Vol. 6, No. 4, Winter 1987.
6. Matsuno, F., Fukushima, S. and Coworkers, "Feedback Control of a Flexible Manipulator with a Parallel Drive Mechanism", International Journal of Robotics Research, Vol. 6, No. 4, Winter 1987.
7. Craig, J.J., "Introduction to Robotics, Mechanics & Control", Addison-Wesley Publishing Company, 1986.
8. Kuo, B.C., "Automatic Control Systems", Prentice-Hall, 1982.
9. Feliu, V., Rattan, K.S., and Brown, H.B., "Model Identification of a Single-Link Flexible Manipulator in the Presence of Friction", 19 Annual Modeling and Simulation Conference, Pittsburgh, May 1988.

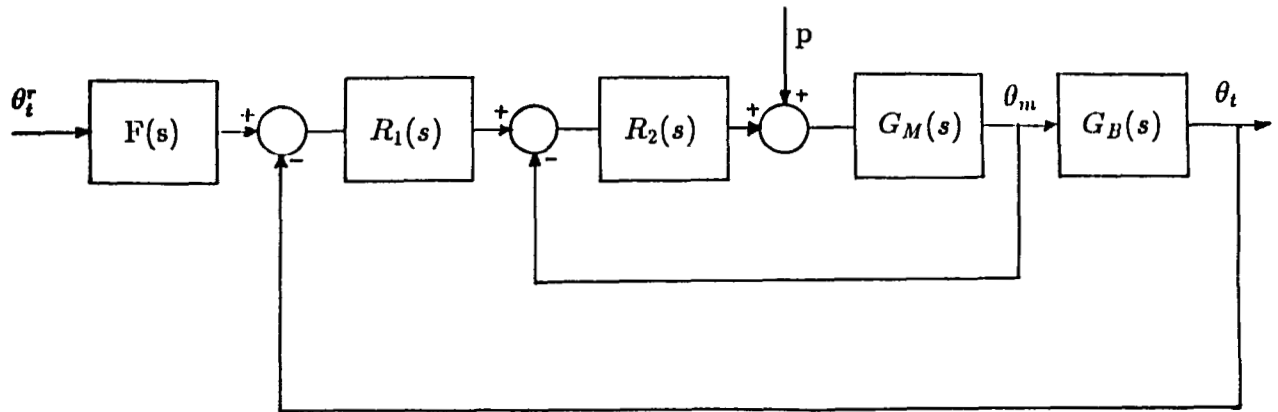


Figure 1. Proposed General Control Scheme.

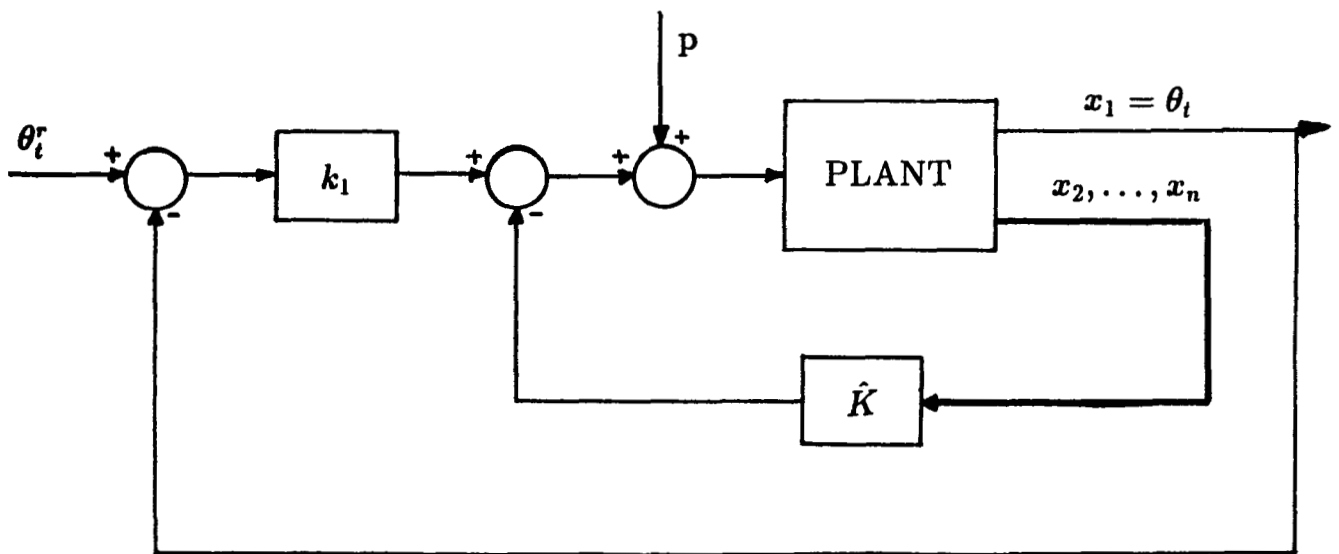


Figure 2. Existing Control Scheme for Flexible Arms.

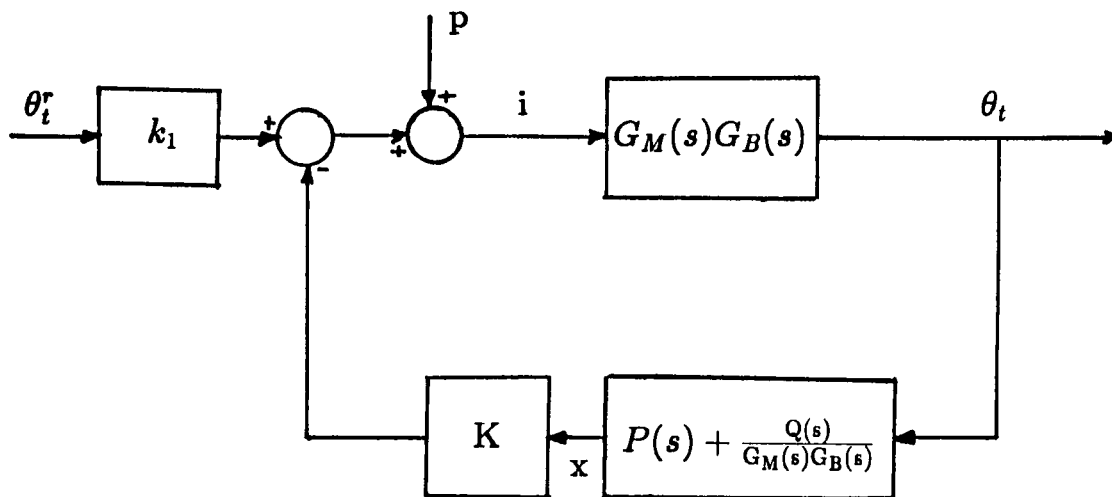


Figure 3. Existing Control Scheme Modified for Output Feedback.

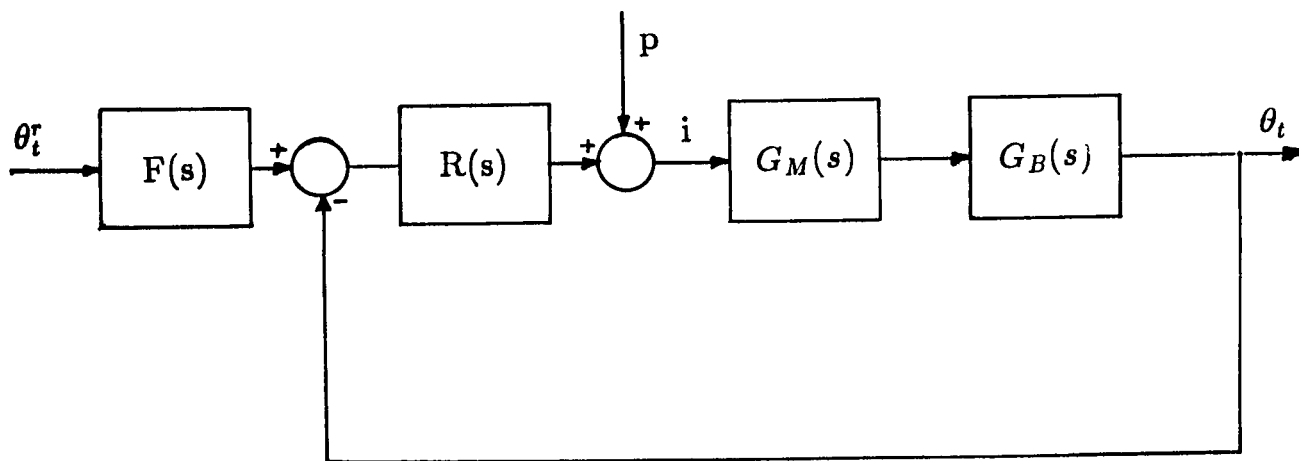


Figure 4. Existing Control Scheme Transformed into Equivalent Transfer Function Form.